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MODELS OF NEW PRODUCT DIFFUSION
CURRENT STATUS AND RESEARCH AGENDA
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Abstract

Diffusion models, mathematical attempts to describe the growth patterns of new products and relate them to variables such as price and population characteristics, have seen major development trends in the past few years. This paper reviews that progress in order to isolate those areas most likely to be fruitful for further development.

The paper first reviews some of the behavioral underpinnings of diffusion models: the phenomena being modeled. Next, the major explicit form models are reviewed and classified according to whether they have market-controls or not and according to the number of stages they explicitly consider. These models are related to larger-scale simulation models (especially in the solar energy area) that use explicit-form models as building blocks.

The paper concludes with a discussion of some important advances that still need to be made in this important and rapidly-growing area.

1. Introduction

Diffusion models are mathematical attempts to relate the adoption rate of a new product to factors including time, population characteristics, and decision variables such as product price. The last several years have seen a growth of interest in this area for several reasons:

- o First, there is increasing concern about a perceived decline in the "innovativeness" of American industry, leading to more analysis of the new product adoption process.

- o Second, product life cycles, particularly in high-technology industries, are continuing to shorten, making forecasting and control early in the life cycle of a new product a most important consideration.

- o Third, as our environment becomes less stable (oil embargos, inflation, fuel shortages, etc.), more sophisticated methods are required to assess the impact of changes in those environmental factors on product adoption rates.

For the above and other reasons, many diffusion models have been developed recently, not all of which are consistent with one another. And, no overall model exists that captures all the relevant phenomena.

The purpose of this paper is first to review the phenomena leading to diffusion processes and then to discuss the major attempts to model those phenomena. This will lead to a research agenda and a set of priorities for further work in this area.

Our immediate focus relates to adoption of alternative energy systems, photovoltaics in particular. However, we feel that adoption of photovoltaics is a special case of a more general process of new product adoption. Thus we study adoption processes in general, and then indicate the specific links to alternative energy systems later.

2. Diffusion Phenomena

A widely studied process in the area of social behavior is what is referred to as diffusion: diffusion of ideas, diffusion of cultural traits, of rumors, of opinions, of fads, of fashions, of diseases, of population itself, and of new technologies. Some early sociologists (Torcle, 1903, for example) made diffusion their central mechanism. Many others have examined empirical cases of diffusion (McVoy, 1940, Kniffen, 1957, Ryan and Gross, 1943, etc.). This empirical work has covered a wide range of topics from hybrid corn (Ryan and Gross) to automobiles (Hagerstrand, 1952) to the practice of boiling drinking water (Wellin, 1955).

From these investigations and many others (Rogers, 1962), a considerable amount of knowledge has developed about the way diffusion in human populations proceeds. For example, diffusion processes differ depending upon the unit of adoption, which may be an individual (new appliance) or an industrial organization (new generation of computers). Or, again, there is evidence that different communications media perform different functions affecting the adoption of an innovation: interpersonal communication plays an influence role, legitimizing a new product, while mass media play a more informative role.

Researchers in the area generally agree the process is complex and most attempts to bring structure to the area have focused on a subset of the following four main elements of the diffusion process:

1. Innovation: what the characteristics of an innovation are that affect the rate and ultimate level of penetration (see Mansfield, 1968, for an investigation of this type).

2. Communications: As noted above, mass media perform different functions in accelerating diffusion than do personal contacts. The role of opinion leadership is important here.
3. The Social System: Different societies have different social structures affecting the adoptive process. In addition, an element that affects diffusion is the incompleteness of social structures: the friends of my friends are also likely to be my friends and that likelihood is far greater than their frequency in the total population would predict. Thus, diffusion may spread only to a small in-group or the diffusion rates within separate groups may be quite different. (Coleman, Katz and Menzel, 1957).
4. Time: A key focus in most diffusion studies is how the rate of diffusion varies over time.

To limit our investigation here, we restrict ourselves to western-type societies where electronic communication is quite effective. Under these circumstances, spatial diffusion, the geographic spread of the diffusion, is a second-order phenomenon. With high-speed travel, electronic communication and mass media, information diffusion is relatively homogeneous in space. (Spatial diffusion though, can be incorporated in diffusion models if necessary.)

Rogers (1962) has reviewed some 500 sources and comes to the following general conclusions about diffusion processes:

1. First purchase of a new innovation follows a bell-shaped curve, and the cumulative penetration follows an S-shaped curve.
2. There are three stages of adoption: awareness, trial (first purchase), and adoption (regular purchase).

3. Salesmen are relatively effective for early adopters while word of mouth becomes more important for late adopters.
4. On average, when dealing with consumers, early adopters are younger, have higher social status and better financial positions than late adopters.

In addition, many studies show that for repeat-purchase items, the most usual pattern of sales is illustrated in Exhibit 1:

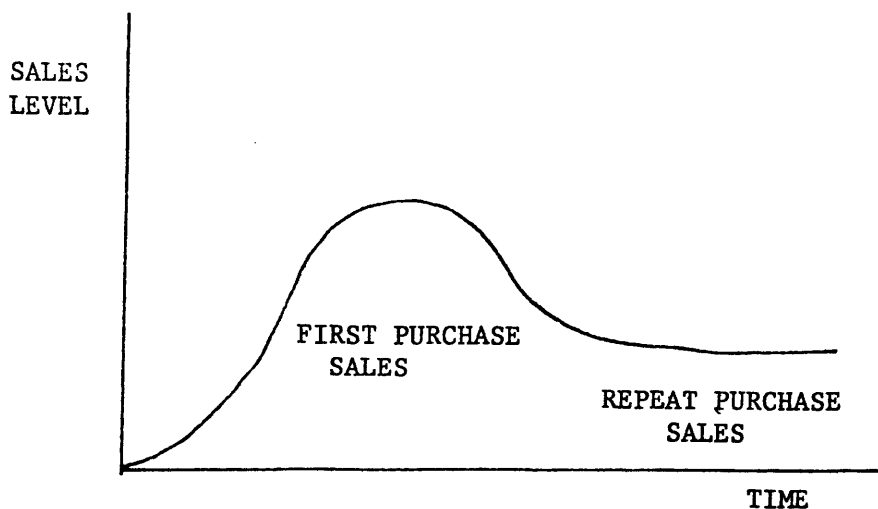


Exhibit 1: Typical pattern for a repeat purchase sales model.

A set of "phenomena" to which little attention has been given relates to controls. The question is, how do advertising, price, promotion and competition affect the diffusion process. Mansfield (1968) gives some clues, indicating that when adoption of a product is more profitable for the adopting company and the cost of adoption is low, the adoption rate will be greater. This suggests that adoption can be affected by

- (a) the price of the product relative to competition, and
- (b) the absolute price of the product.

Given this background, some of the characteristics we would like a general model of new product adoption to have include:

1. The cumulative number of first purchases should be a (generally) S-shaped, monotonically increasing function with an upper bound. For repeat-purchase-type products, cumulative, first-purchase sales should follow the same sort of pattern as that above.
2. A general model of first purchase should have at least three states: unaware, aware, and adopter.
4. A repeat-purchase-type model should have four states: unaware, aware, adopter, past adopter.
5. Marketer-controls should be included in the model. For example, advertising will affect the flow from unawareness to awareness as well as from awareness to first purchase. Price should affect the flow from awareness to purchase.
6. Whether the focus of study is a generic product adoption or brand adoption, the effect of competition (substitutable product) should be included if possible.

3. Diffusion Models

3.1 Criteria for Evaluation

Clearly, diffusion processes are complex phenomena. Mathematical models designed to incorporate all those phenomena might be hopelessly complex, however complete they are. The model-building process calls for a balance between theory, data, and the use for which the model is intended.

Several sets of criteria have been developed to understand and evaluate the model-building process. Lilien (1975) suggests that models should represent different levels of complexity depending upon the use as well as the user. For example, a model aimed at sales forecasting for the purpose of inventory control may be adequate for the operations department, but useless for the advertising department interested in advertising evaluation.

Little (1970) discusses some criteria for evaluating models. To be useful for a manager, he suggests a model should be

- o Simple--understandable by the user
- o Robust--absurd answers are difficult to obtain
- o Easy to control--amenable to manipulations that provide easy analysis of model sensitivity
- o Adaptive--capable of being updated as more data become available
- o Complete--including all the most important variables
- o Easy to communicate with.

All the models we will review here make explicit or implicit trade-offs in these criteria. Models in this area also can be separated into two groups:

- I. General, Explicit-Form Models: These models explicitly state all the relationships between the variables, generally in one or a few equations. They are not developed to apply to any specific technology.
- II. Simulation (Total System) Models: These models, developed to incorporate more detail than the general models, usually are restricted to specific types of innovations (solar products, for example) and have specific objectives (forecasting vs policy evaluation). They often include aspects of the general models above as specific equations.

There have been many developments of simulation models, particularly in the solar energy area. Warren (1979) reviews many of the most important efforts, including those of MITRE's (1977) SPURR model, SRI's (1978) model, Midwest Research Institute's (1978) model, A.D. Little's (1978) model, and Energy and Environmental Analysis' ISTUM (1978) model. Warren (1979) notes that, with respect to models in this area,:

Valid data can be entered in a spurious model to yield meaningless numbers. This appears to be the case with solar energy market penetration analysis. Thus . . . in their present form, solar energy market diffusion models are closely akin to number mysticism (p. 25).

Lilien's (1978) PVO model is an attempt to bring a sounder behavioral structure to bear on the problem. That model uses the current theory of organization adoption of new products and is calibrated with field measurements. It shows many of the weaknesses of other simulation models, but does explicitly include a basic diffusion-model structure. This is clearly a step in a direction that Warren (1979) endorses:

. . . it is recommended that standard diffusion models be used in solar energy market penetration analyses . . .
This would at least put the analyses in a sound diffusion

model framework so that their theoretical bases could be analyzed and compared. However, in order to be able to place reasonable confidence in the result of solar energy market penetration models, a market penetration analysis based on behavioral relationships must be developed . . . Building such a model is a prerequisite for solar energy market penetration modeling to leave the realm of numerology (p. 26).

The message here is clear: for us to make progress in analyzing new product diffusion, even through simulation models, a sound understanding of the basic diffusion models, the building blocks of the simulation models, is required. Our objective here, then, is to review the most advanced of the Explicit Form Models and indicate where work in this area, in terms of synthesis and model development, is needed. These models, in turn, will provide a sounder base for future simulation models.

In reviewing the Explicit-Form models, we must address the trade-off between model use and theory-representation. A complete model incorporating all the important phenomena may be as "true" as possible, and it may not be capable of being tested or used. The data required to estimate its parameters may be either unavailable or be unable to be generated. Clearly, as we move to more complete models, we will have more data, estimation, and interpretation problems.

We make one final distinction between first-purchase models and repeat-purchase models. Although repeat-purchase models with controllable variables have been developed in a diffusion model framework (Lilien, Rao and Kalish, 1979), few examples exist. Our focus here will therefore be mainly on single-purchase models.

3.2 General, Explicit-Form Models

Most of the general, explicit-form models consider only two stages: those who have purchased and those who have not. We review the models according to the structure in Exhibit 2.

NUMBER OF MODEL STAGES

2 STAGES

3 STAGES

NO CONTROL
VARIABLES

BASS 1969
 JEULAND
 MAHAJAN and PETERSON

WITH CONTROL
VARIABLES

BASS 1978
 ROBINSON and LAKHANI
 DOLAN and JEULAND
 JEULAND
 HORSKY and SIMON

DODSON and MULLER

EXHIBIT 2: MODEL CLASSIFICATION

3.2.1 Two-Stage Models with No Control Variables

Many simple models have been developed, modeling a two-stage diffusion process without control variables (see Mahajan and Peterson, 1979, for a review). Two-stage models without controls are simple: their obvious advantage is simplicity and easy estimation. However, as discussed later, we feel the phenomena are best described with a three-stage model. And, without control variables, the model's decision-making use is limited. However, they do reproduce the S-shaped curve for cumulative adopters under certain conditions.

Most two-stage models assume that the rate of adoption is proportional to the number of people who have not yet adopted. Let $N(t)$ be the eventual number of adopters, and $X(t)$ the cumulative number of adopters. This assumption means:

$$(1) \quad \dot{X}(t) = f(X(t)) \cdot (N(t) - X(t))$$

Bass (1969) in one of the best known models of this type, assumes that the likelihood of adoption is a linear function of the number of current adopters, i.e.:

$$(2) \quad f(X) = a + b \cdot X,$$

So his model is

$$(3) \quad \dot{X} = (a + b \cdot X)(N - X).$$

As Jeuland (1979) points out, it is unclear what Bass means by "likelihood." If he means individual likelihood then we have a time-homogeneous birth process with transition probability

$$(4) \quad \lambda(X) = (N - X)(\alpha + \beta X)$$

and the expected number "born" through this process is different from the solution of the deterministic equation above.

Thus it is critical to specify assumptions clearly; here we will consider the rate of adoption at the aggregate level.

The solution to the Bass model is

$$(5) \quad X(t) = N \frac{1 - e^{-(a+bN)t}}{1 + e^{-(a+bN)t} \cdot \frac{bN}{a}}$$

(assuming N constant and starting with no previous adopters at time 0).

The interpretation given to the parameters a and b is the following: a is the coefficient of innovation, i.e., it is a constant proportion of the population that is not influenced by others; b is the coefficient of imitation, so as the number of adopters, X , goes up, so does their influence (bX) on the remaining population not currently adopting. Note that the model proposed by Fourt and Woodlock (1960) is a special case where $b = 0$. The models proposed by Mansfield (1968), Blackman (1974), and Fisher and Pry (1971) all assume $a = 0$.

When $b > a$ and the word-of-mouth effect is dominant, the solution to the equation has an S-shape. But when $a > b$, and innovative behavior is dominant, the rate of adoption is decreasing. Exhibit 3 shows cumulative adoption for these two cases.

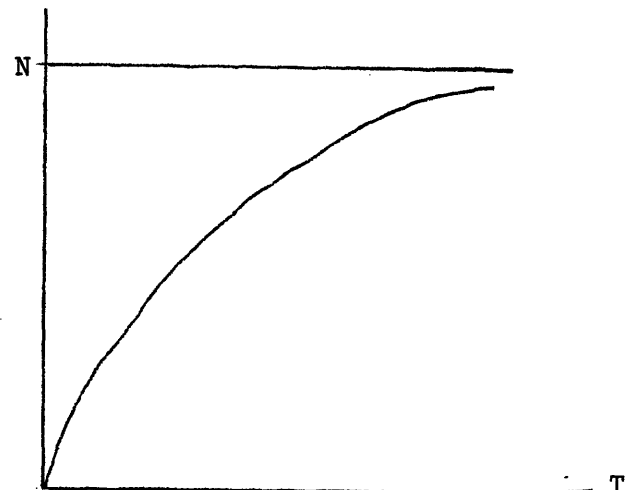
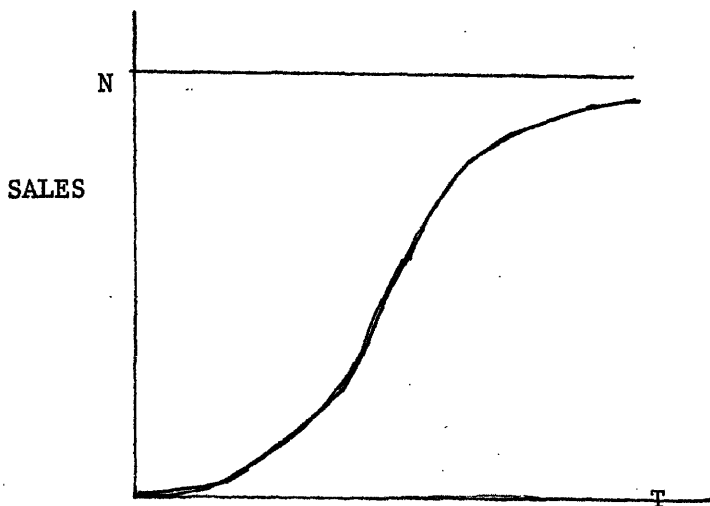


Exhibit 3 Cumulative Adoption in the Bass Model

Although the model is overly simple (as are all two-stage models) and has no controls, it does seem to fit past data well. Thus as a forecasting and descriptive tool it appears to be quite useful.

A drawback is the assumption that the population is homogeneous; clearly some people are more innovative than others. If innovativeness is correlated with income, education, etc., one would expect these innovators to adopt early and the proportion of innovative people would decrease in the nonadopting group. This suggests that the parameter, a , must go down in time. A step in this direction has been made by Jeuland (1979) who incorporates income into the coefficient of innovation. If we look at group i with income I , let

$$(6) \quad a = a_0 + a_1 \cdot I.$$

If we assume that this group is homogeneous then for this group we have:

$$(7) \quad \frac{dX_i}{dt} = (N_i - X_i)(a_0 + a_1 \cdot I_i + bX), \text{ where } i \text{ is an income-group index.}$$

Letting the income density distribution be gamma with parameters α and β , we integrate over the whole population and we get:

$$(8) \quad X(t) = N \frac{1 - e^{-(bN - a_0)t} (1 + \beta a_1 t)^{-\alpha}}{1 + bN \int_0^t e^{-u[bN - a_0]} (1 + \beta a_1 u)^{-\alpha} du}$$

and penetration now is also a function of socioeconomic parameters. Jeuland does not discuss applications or how to resolve estimation problems, however.

Following upon this point, most models to date have underplayed estimation problems. To develop a statistically sound estimation method,

we must explicitly consider the stochastic nature of the process. Bass uses a discrete analog of (7):

$$(9) \quad \Delta X(t) = A + B \cdot X(t-1) + C X^2(t-1), \text{ where}$$

$$(10) \quad A = aN, B = [b \cdot N - a], C = b$$

Several problems arise here. One is that in most instances, X and X^2 will be highly correlated. This, combined with the small sample available for estimation, results in multicollinearity in B and C . Another is that errors tend to be autocorrelated in this structure. And finally, unless these (least squares) estimates are maximum likelihood estimates, a , b , and N are likely to be inconsistent. Clearly, some attention needs to be paid to the problem of estimation.

An extension to Bass' model has been developed by Mahajan and Peterson (1978). They let the potential adopter population be a function of time, i.e., $N = N(t)$. This is especially important for products whose life cycle is very long. They give an example where housing starts influence the potential market, and report good results.

3.2.2 Two-Stage Models with Control Variables

Most of the models that incorporate controls are extensions of the basic Bass Model. We review the Horsky and Simon (1978) model (which incorporates advertising effects) and Bass (1978), Robinson and Lakhani (1975), Dolan and Jeuland (1979) and Jeuland (1979), who incorporate price. The results are encouraging but require further refinement.

3.2.3 Models Incorporating Advertising

Horsky and Simon (1978), while recognizing that price affects the eventual number of adopters - N - rather than the coefficients a and b , only incorporate the effect of advertising in their model. Advertising

makes people aware of the product, and influences purchase. Therefore its primary effect will be on direct buying--the coefficient of innovation. So they let the coefficient of innovation be a linear function of advertising spending:

$$(11) \quad a = a_1 + a_2 \times A(t), \text{ where}$$

$A(t)$ = advertising spending at time t .

The model makes sense only locally since the implication of the linear model is that the rate of purchase can be increased without bound by increasing advertising. Thus this model might be a good descriptive tool, but it is not aimed at developing optimal policies.

Horsky and Simon tested their model and a_2 --the coefficient of advertising--turned out significant in three out of five cases. This model, a step in the right direction, could be improved by including a more robust advertising relationship. Such a relationship would be nonlinear and have "memory." One way to model this is to let

$$(12) \quad a(t) = \int_0^t r(A(t)) e^{-\gamma t} dt.$$

where $\frac{r(A(t))}{\gamma}$ is the value a would take under a long-term policy of constant advertising at the rate $A(t)$ and γ is a forgetting rate.

3.2.4 Models Incorporating Price

Robinson and Lakhani (1975) criticize the use of conventional pricing methods for rapidly growing businesses, for two reasons: the first is that cost goes down rapidly. The other is that demand behaves according to a diffusion model rather than being static.

Let

$$Q(t) = Q_0 + \int_0^t v \, dt$$

where

$Q(t)$ = cumulative volume sold by t and

V = sales rate at t

P = price per unit,

C = cost per unit of production,

δ = discount rate.

Then they suggest that the appropriate profit to maximize is the present value of planning period profit:

$$(13) \quad \max_{p(t)} \pi = \int_0^{t_F} [P - C] V e^{-\delta t} dt.$$

where

t_F = end of planning period

$V = V[P, Q, t]$

$C = C[V, Q, t]$

They assume that cost goes down according to the learning curve:

$$(14) \quad C = C_0 \cdot \left[\frac{Q_0}{Q} \right]^\alpha$$

(Boston Consulting Group (1970)).

Demand is assumed to behave according to Bass' (1969) model. i.e.,

$$(15) \quad V(t) = a[Q_m - Q] + b[Q_m - Q] \cdot Q$$

$$= A \left[1 - \frac{Q}{Q_m} \right] \cdot \left[B + \frac{Q}{Q_m} \right]$$

where

$$A = b \cdot Q_m^2, \quad B = \frac{a}{bQ_m}.$$

and Q_m = market potential.

Robinson and Lakhani suggest that A should not be constant but a function of price and advertising, etc., so that the penetration rate can be related to economic variables.

As an example they use

$$(16) \quad V \text{ proportional to } e^{-\beta P}$$

and give an illustration. They solved the problem for an optimal profit policy and showed that this policy was superior to many others. Because they do not present closed form results however, it is not clear how general these results are.

Bass (1978) tries to analyze a similar model. He assumes a cost structure according to the learning curve, but assumes constant price elasticity η . He also assumes that the firm prices to maximize current profit, i.e.:

$$(17) \quad P(t) = \frac{\eta_t}{\eta_t - 1} mc = \left[\frac{\eta_t}{\eta_t - 1} \right] \cdot C_0 \left[\frac{Q_0}{Q} \right]^\alpha$$

where mc = marginal cost of production, and demand. Demand, V ., is assumed to be of constant elasticity, but shifted in time by $f(t)$;

$$(18) \quad V(t) = f(t) \cdot C \cdot [P(t)]^{-\eta}$$

So if we substitute (17) in (18) we get

$$(19) \quad V = \dot{Q} = f(t) \cdot K \cdot Q^{\alpha\eta}$$

which has the following solution:

$$(20) \quad Q(t) = [K (1 - \eta\alpha)]^{\frac{1}{1-\eta\alpha}} F(T)^{\frac{1}{1-\eta\alpha}}.$$

where $F(t) = \int_0^t f(t) dt$.

And if we write

$$m = \lim_{T \rightarrow \infty} Q(T) = [K(1 - \alpha\eta)]^{\frac{1}{1-\alpha\eta}}$$

we get

$$(21) \quad V(t) = \frac{m}{1 - \alpha\eta} f(t) \cdot F(t)^{\frac{\alpha\eta}{1-\alpha\eta}}$$

and

$$(22) \quad P(t) = \left[\frac{\eta}{\eta - 1}\right] \cdot C_0 m^{-\alpha} F(t)^{-\frac{\alpha}{1-\alpha\eta}}.$$

Note that $\dot{P} < 0$, which implies that price decreases monotonically.

Clearly this is a descriptive tool, as Bass assumes instantaneous profit maximizing behavior. It is not clear that this is a reasonable objective so the results have limited normative use. Bass does some rough empirical testing of the model and gets reasonable results.

Dolan and Jeuland (1979) use the same model as Robinson and Lakhani did and get more analytical results. First they assume nonstatic demand as follows:

$$(23) \quad \max_{P(t)} \int_0^{t_F} [P - C] V e^{-\delta t} dt$$

$$V = a e^{-d P(t)}$$

$$C = C_0 \left[\frac{Q_0}{Q}\right]^{+\alpha}$$

$$C, V, P > 0.$$

This has an analytical solution:

$$(24) \quad P^*(t) = \frac{1}{d} + \frac{C_0}{Q^{-\alpha}} e^{-\lambda(t-t_F)} [V(t_F)]^{-\alpha} + \lambda e^{\lambda t} (g(t))$$

where

$$g(t) = \int_t^{t_F} C e^{-\lambda z} [V(z)]^{-\alpha} dz$$

is the discounted total cost of production incurred from t on. Note that for the case of no discounting, the optimal price is constant and depends only on the final cumulative volume. With discounting, the optimal price is monotonically decreasing.

When they incorporate the demand according to diffusion models, the authors get analytical results only for the case of no discounting and a finite planning horizon. Their results take the shape displayed in Exhibit 4.

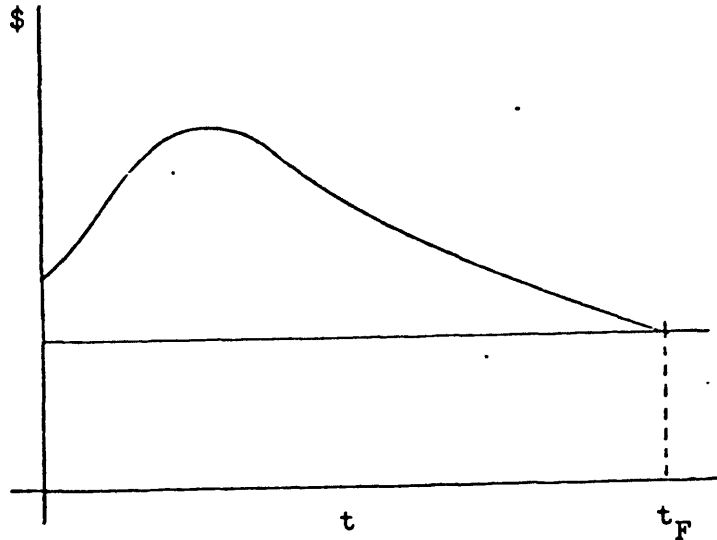


Exhibit 4. Optimal pricing behavior:
dynamic demand, finite planning horizon, no discounting.
(Dolan and Jeuland (1979)).

This pattern shows the optimal pricing policy is to start with a low price in order to accelerate the diffusion process, and to get costs down

fast, then increase the price and finally monotonically decrease the price to

$$\frac{1}{d} + c[Q(t)]^{-\alpha}$$

To summarize we note

1. Robinson and Lakhani's formulation is general, for a dynamic situation. Dolan and Jeuland used a specific demand and cost structure, to get some nice analytical results. More work may yield results in the discounted dynamic demand situation. Also, other cost and demand structures may be explored, as the learning curve behavior in the long run--where costs approach zero--is questionable.

2. One of the drawbacks of all the models in this sections is--as Jeuland (1979) points out--that they assume the following demand structure:

$$(25) \quad \dot{X} = (N - X)(a + b \cdot X) \cdot h(p)$$

where $h(p)$ is the function that incorporates the price effect. The implication of such models is that the market potential is the same-- N --and is not influenced by price. Only the rate of adoption is influenced. Another implication is that the effect of price is the same for innovators and for imitators. Both implications are questionable. Jeuland suggests another formulation, which is appealing, but rather complicates the model.

3.2.4 Three-Stage Models

Three-stage models give the more realistic view of a state of awareness prior to adoption. This is a trade-off with simplicity, and the ability to control and estimate parameters. The model that will be described here is the one introduced by Dodson and Muller (1978). Other

three-stage models have been introduced (Gould, 1970, Bernhard and Mackenzie 1972), but this is the most general one. First we review the model for one-time purchases and then develop the extension to repeat purchases. The concept is described in Exhibit 5.

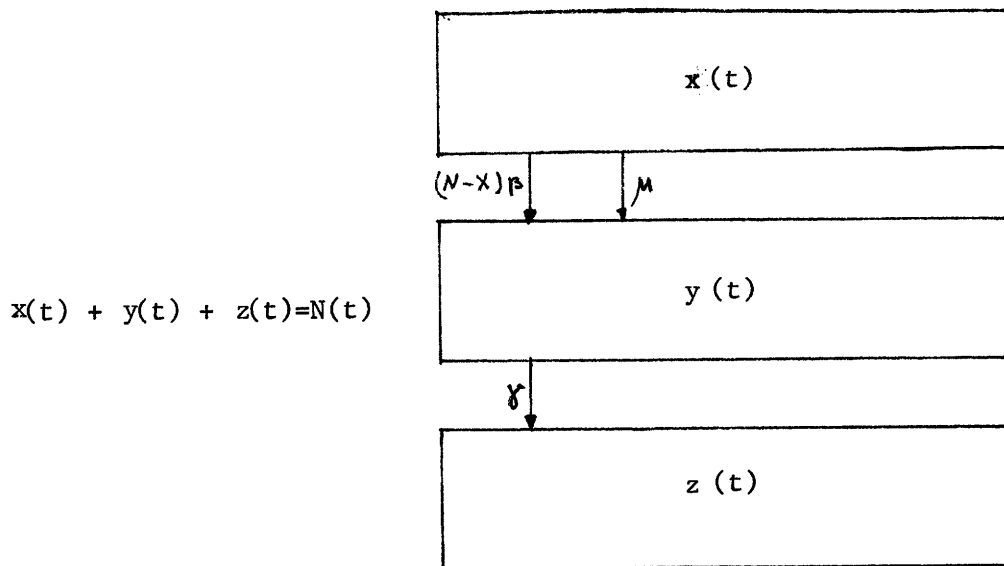


Exhibit 5. Three-Stage Adoption Model

The total potential population $N(t)$ is divided into three groups:

$x(t)$ – Those who are unaware of the product

$y(t)$ – Those who are aware, but did not adopt yet

$z(t)$ – Those who have adopted already.

The flow from unawareness to awareness is governed by two factors:

1. A word-of-mouth effect
2. Advertising and promotion.

The flow from awareness to adoption is also governed by two factors:

1. Imitation phenomena
2. Independent phenomena like price, advertising, and

innovativeness.

Dodson and Muller simplify the second flow and assume it has a constant rate.

The sales rate is the increase in adoption per period, i.e., z and the state equations are:

$$(26a) \quad \dot{x} = \beta x(y + z) - \mu x$$

$$(26b) \quad \dot{y} = \beta x(y + z) + \mu x - \gamma \cdot y$$

$$(26c) \quad \dot{z} = \gamma y$$

Dodson and Muller show that several models, including Bass' (1969), are special cases of this model.

An extension of the above formulation to include repeat-purchase is as follows:

- A constant proportion, ϕ , of the $(y + x)$ forgets about the new product, and flows back to the unawareness stage.
- Only a portion, γ , of current buyers will buy next period, due to competition.
- A proportion, θ , of z switches brands.

The state equations then become:

$$(27a) \quad \dot{x} = \beta x(N - x) - \mu x + \phi(N - x)$$

$$(27b) \quad \dot{y} = \beta x(N - x) + \mu x - (\gamma + \phi) \cdot y + \theta z$$

$$(27c) \quad \dot{z} = \gamma y - (\phi + \theta)z$$

And the sales are given by

$$(28) \quad s(t) = \gamma y + \bar{\gamma} z, \text{ where } \gamma \text{ is the repeat-purchase parameter.}$$

A closed-form solution of the above equations is not possible, but Dodson and Muller give some characterization of the solution, through the use of phase diagrams.

Their main result is the following: the solution to (27) is monotonically increasing in the number of informed people, $N - x$, and the number of current customers z , and is either monotonically increasing or single-peaked in y , and sales, S .

Assuming different relationships between the parameters, the model produces different shapes of sales versus time, consistent with the different phenomena described earlier.

An interesting special case is when the trial rate equals the repurchase rate, i.e., $\gamma = \bar{\gamma}$, and $\beta = \bar{\theta}$, $\theta = 0$. This represents the case where awareness is generated only through advertising, and there is no "forgetting." For this case, sales are:

$$(29a) \quad s(t) = \gamma(N - x(t))$$

or

$$(29b) \quad \dot{s}(t) = \gamma(N - \dot{x}(t))$$

and after substituting for x we get:

$$(30) \quad \dot{s}(t) = \mu[\gamma N - \dot{s}(t)] - \phi s(t)$$

Recall that μ represents the flow from awareness to unawareness, due to advertising. If we model μ linearly, the above turns out to be the Vidale-Wolfe model (1957). The models of Gould (1970), Nerlove and Arrow (1962) and Glaister (1974) are also shown to be special cases.

In order for the model to be of managerial use, we have to be able to estimate parameters. The authors suggest the following procedure. If we write the equation for sales as

$$(31) \quad s(t) = a + b x(t) + c y(t)$$

where

$$a = \bar{\gamma}N, \quad b = -\bar{\gamma} \quad \text{and} \quad c = \gamma - \bar{\gamma}$$

then the above parameters can be estimated by least-squares regression. Unfortunately, the data for $x(t)$ and $y(t)$ are seldom available.

An alternative procedure, suggested by the authors, is to search over an exhaustive list of parameters, and choose that set that best reproduces sales data. The authors report using this procedure and getting good results.

This three-stage model, more realistic than the two-stage model, requires further development:

1. Controls should be included explicitly: μ should be modeled to include the effect of advertising and the effect of price, in the flow from stage 2 to 3.
2. Work is needed in parameter estimation and updating.
3. Time-varying parameters should be included.
4. The repeat purchase process should be extended to include four stages.

4. Assessment and Directions for Further Work

Our review of the status of this field suggests that much more development is required before convergence occurs. (By convergence, we mean a general agreement about the appropriate approach and the appropriate model structure in any given situation.)

In particular, the key challenge is to build a more complete and more general model, incorporating the phenomena noted in Section 2. This can be done best, as a first step, for first-purchase models. To move in that direction, the following research issues need to be addressed:

1. Structure of First Purchase, Product Class Models:

We feel that the three-stage, Dodson and Muller-type model is most appropriate. Research is needed on how control variables influence the flow from unawareness to awareness and from awareness to purchase. How (when) does advertising, personal selling, pricing work here?

2. Time-Varying Parameters:

The innovation rate should go down and the imitation rate should go up with time. Investigation on how to incorporate the effect in the model is needed.

3. Estimation and Data Collection:

None of the existing models presents a rigorous treatment of issues of estimation in a diffusion model framework. The problem is serious: little or no data and multicollinearity affect the problem precisely when the information from the model is most valuable. Bayesian estimation and updating of parameters should be investigated as a framework for developing and updating policies that are to be used early in the life cycle of the product.

4. Stochastic Adoption Processes:

Current models are currently (a) deterministic and (b) sensitive to small changes in some of the deterministic inputs (the level of the learning parameter in a cost decline function, for example). We should investigate the inclusion of the stochastic nature of the process explicitly, with utility maximization as an objective.

5. Government Policy Trade-Offs:

For alternative energy systems like photovoltaics, the government faces the problem of determining the level and timing of direct subsidies and demonstration programs to meet certain objectives. Government action embedded in a diffusion model assuming optimal industry behavior should be investigated.

Two other issues that might be investigated include (a) brand-diffusion—the study of new product diffusion by a brand within a product class and (b) an extension of the above ideas to repeat-purchase models. A four-stage model including aware and unaware purchasers and past purchasers would be appropriate here.

In sum, important early work has been done in this area. What faces the researcher now is the task of integration and consolidation, balancing models with use, estimation with data limitations. Much of this key integrating work should be done in the next few years, providing new insight into the process as well as new tools to reduce the risks and maximize the success rates for product innovations.

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